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TRACKING OF MULTIPLE MANEUVERING TARGETS IN CLUTTER USING MULTIPLE SENSORS, IMM AND JPDA COUPLED FILTERING

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ABSTRACT

We consider the problem of tracking multiple maneuvering targets in clutter using switching multiple target motion models. A novel suboptimal filtering algorithm is developed by applying the basic interacting multiple model (IMM) approach, the joint probabilistic data association (JPDA) technique and coupled target state estimation to a Markovian switching system. In past such an approach has been considered using uncoupled target state estimation. The algorithm is illustrated via a simulation example involving tracking of two highly maneuvering, at times closely spaced, targets. In the presented example, the proposed IMM/JPDA coupled filter outperforms an existing IMM/JPDA uncoupled filter.

1. INTRODUCTION

We consider the problem of tracking multiple maneuvering targets in clutter. This class of problem has received considerable* attention in the literature [3],[6],[7], [13], [14]. The switching multiple model approach has been found to be quite effective in modeling highly maneuvering targets [1], [3], [5]-[8], [12]. In this approach various "modes" of target motion are represented by distinct kinematic models, and in a Bayesian framework, the target maneuvers are modeled by switchings among these models controlled by a Markov chain. In the presence of clutter, the measurements at the sensors may not all have originated from the target-of-interest. In this case one has to solve the problem of data association. An effective approach in a Bayesian framework is that of probabilistic data association (PDA) [3], [5] for a single target in clutter and that of joint probabilistic data association (JPDA) [3], [6], [13] for multiple targets in clutter.

It is assumed that the number of targets is known (say N) and that for each target, a track has been formed (initiated) and our objective is that of track maintenance. In [15] such a problem has been considered for a single target using multiple sensors, PDA and switching multiple models. The optimal solution (in the minimum mean-square error sense) to target state estimation given sensor measurements and absence of clutter, requires exponentially increasing (with time) computational complexity; therefore, one has to resort to suboptimal approximations. For the switching multiple model approach, the interacting multiple model (IMM) algorithm of [8] has been found to offer a good compromise between the computational and storage requirements and estimation accuracy [16]. In the presence of clutter, one has to account for measurements of uncertain origin (target or clutter?). Here too, in a Bayesian framework, one has to resort to approximations to reduce the computational complexity, resulting in the PDA filter [12], [3], [6], [2], [15]. In [15] the IMM algorithm has been combined with the PDA filter in a multiple sensor scenario to propose a combined IMM/MSPDAF (interacting multiple model/ multisensor probabilistic data association filter) algorithm. In [3], [13] and [14] multiple targets in clutter (but without using switching multiple models) have been considered using JPDA filter which, unlike the PDA filter, accounts for the interference from other targets. Various versions of IMMJPDA filters for multiple target tracking using switching multiple models may be found in [4], Sec. 6 of [6], [10] and [11]. While [10] and [11] present uncoupled filters (i.e. assume that different target states are mutually independent conditioned on the past measurements), [4] and [6] present IMMJPDA coupled filters where the conditional target state independence assumption is not made. This assumption is false when the targets are closely spaced thereby "sharing" measurements (Sec. 6 of [6]).

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It has been noted in [9] that the IMMJPDA coupled filter equations of [4] and [6] are heuristic. [9] presents an "exact" JPDA coupled filter for non-switching models using the framework of a linear descriptor system with stochastic coefficients.

In this paper we extend the approach of [10] which pertains to uncoupled filtering, to IMMJPDA coupled filtering. The only approximations made are those typical for IMM approaches; there are no other heuristics as in [4] and [6]. Furthermore, we use the standard Markovian switching state-space systems; unlike [9], a linear descriptor system framework is not necessary.

In this paper we exploit the basic structure of [15] in combination with the JPDA algorithm of [3] and [13] to propose a novel IMM/JPDA coupled filtering algorithm. As in [15] it is assumed that the sensors are collocated and (time) synchronized with the sampling rate. Track initialization (formation) is assumed to have been made for each target. "Standard" assumptions are used for JPDAF ([13], p. 310 of [6]): a measurement can have only one source; among the possibly several validated measurements, at most one of them can be target-originated and the remaining validated measurements are assumed to be due to false alarms or clutter, and are modeled as independently and identically distributed (i.i.d.) with uniform spatial distribution over the entire validation region ("across all targets"). Also, as in [15] and [14], we use sequential updating of the state estimates with measurements (i.e. updating the state estimates sequentially with measurements from different sensors).

2. PROBLEM FORMULATION

Assume that there are total N targets with the target set denoted as $\mathcal{T}_N := \{1, 2, \cdots, N\}$. Assume that the dynamics of each target can be modeled as one of the n hypothesized models. The model set is denoted as $\mathcal{M}_n := \{1, 2, \cdots, n\}$ and there are total q sensors. For target r ($r \in \mathcal{T}_N$), the event that model i is in effect during the sampling period $(t_{k-1}, t_k]$ will be denoted by $M_k^i(r)$. Although all the targets share a common model set, any two targets may be in different motion status from time to time.

For the j-th hypothesized model (mode), the state dynamics and measurements of target r ($r \in \mathcal{T}_N$) are modeled as

$$x_k(r) = F_{k-1}^j(r)x_{k-1}(r) + G_{k-1}^j(r)v_{k-1}^j(r) \tag{1}$$

and

$$z_k^l(r) = h^{j,l}(x_k(r)) + w_k^{j,l}(r) \quad \text{for} \quad l = 1, \cdots, q \tag{2}$$

where $x_k(r)$ is the system state of target r at t_k and of dimension n_x (assuming all targets share a common state space), $z_k^l(r)$ is the (true) measurement vector (i.e. due to target r) from sensor l at t_k and of dimension n_{zl}, $F_{k-1}^j(r)$ and $G_{k-1}^j(r)$ are the system matrices when model j is in effect over the sampling period $(t_{k-1}, t_k]$ for target r and $h^{j,l}$ is the nonlinear transformation of $x_k(r)$ to $z_k^l(r)$ ($l = 1, 2, \cdots, q$) for model j. A first-order linearized version of (2) is given by

$$z_k^l(r) = H_k^{j,l}(r)x_k(r) + w_k^{j,l}(r) \quad \text{for} \quad l = 1, \cdots, q \tag{3}$$

where $H_k^{j,l}(r)$ is the Jacobian matrix of $h^{j,l}$ evaluated at some value of the estimate of state $x_k(r)$ (see Sec. 3.). The nature of the system state, the various matrices in (1) and (3), and the measurements is specified in more detail in Sec. 4.. The process noise $v_{k-1}^j(r)$ and the measurement noise $w_k^{j,l}(r)$ are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices Q_{k-1}^j (same for all targets) and $R_k^{j,l}$ (same for all

targets), respectively. At the initial time t_0, the initial conditions for the system state of target r under each model j are assumed to be Gaussian random variables with the known mean $\bar{x}_0^j(r)$ and the known covariance $P_0^j(r)$. The probability of target r in model j at t_0, $\mu_0^j(r) = P\{M_0^j(r)\}$, is also assumed to be known. The switching from model $M_{k-1}^i(r)$ to model $M_k^j(r)$ is governed by a finite-state stationary Markov chain (same for all targets) with known transition probabilities $p_{ij} = P\{M_k^j(r)|M_{k-1}^i(r)\}$. Henceforth, t_k will be denoted by k.

In coupled state estimation the states of all targets are estimated jointly [6]. To this end define the "global" state

$$x_k := \text{col}\{x_k(1), x_k(2), \cdots, x_k(N)\} \tag{4}$$

and the corresponding matrices/vectors

$$J := \text{col}\{j_1, j_2, \cdots, j_N\}, \quad j_m \in \mathcal{M}_n \text{ is model } j \text{ for target } m, \tag{5}$$

$$F_k^J := \text{block}-\text{diag}\left\{F_k^{j_1}(1), F_k^{j_2}(2), \cdots, F_k^{j_N}(N)\right\}, \tag{6}$$

$$G_k^J := \text{block}-\text{diag}\left\{G_k^{j_1}(1), G_k^{j_2}(2), \cdots, G_k^{j_N}(N)\right\}, \tag{7}$$

$$v_k^J := \text{col}\left\{v_k^{j_1}(1), v_k^{j_2}(2), \cdots, v_k^{j_N}(N)\right\}. \tag{8}$$

Then we have the state equation for the N targets as

$$x_k = F_{k-1}^J x_{k-1} + G_{k-1}^J v_{k-1}^J \tag{9}$$

where $E\{v_k^J v_k^{J\prime}\} = Q_k^J := \text{block}-\text{diag}\left\{Q_k^{j_1}, \cdots, Q_k^{j_N}\right\}$. Similarly define the global measurement vector at sensor l as

$$z_k^l := \text{col}\left\{z_k^l(1), z_k^l(2), \cdots, z_k^l(N)\right\} \tag{10}$$

and the corresponding vectors

$$h^{J,l}(x_k) := \text{col}\left\{h^{j_1,l}(x_k(1)), \cdots, h^{j_N,l}(x_k(N))\right\}, \tag{11}$$

$$w_k^{J,l} := \text{col}\left\{w_k^{j_1}(1), w_k^{j_2}(2), \cdots, w_k^{j_N}(N)\right\} \tag{12}$$

where $E\{w_k^{J,l} w_k^{J,l\prime}\} = R_k^{J,l} := \text{block}-\text{diag}\left\{R_k^{j_1}, \cdots, R_k^{j_N}\right\}$. Then the measurement equation for N targets at sensor l (assuming no clutter and perfect detections) is given by

$$z_k^l = h^{J,l}(x_k) + w_k^{J,l} \quad \text{for} \quad l = 1, \cdots, q. \tag{13}$$

Following (3), a first-order linearized version of (13) is given by

$$z_k^l = H_k^{J,l} x_k + w_k^{J,l} \quad \text{for} \quad l = 1, \cdots, q \tag{14}$$

where

$$H_k^{J,l} := \text{diag}\left\{H_k^{j_1,l}(1), H_k^{j_2,l}(2), \cdots H_k^{j_N,l}(N)\right\}, \tag{15}$$

Following (5), define the global mode

$$M_k^J := \{M_k^{j_1}(1), \cdots, M_k^{j_N}(N)\}. \tag{16}$$

The various targets are assumed to evolve independently of each other. Therefore, the transition probability for the global modes are given by

$$p_{IJ} := P\{M_k^{j_1}(1), \cdots, M_k^{j_N}(N) \,|\, M_{k-1}^{i_1}(1), \cdots, M_{k-1}^{i_N}(N)\}$$

$$= \prod_{l=1}^{N} p_{i_l j_l}. \tag{17}$$

Similarly we have

$$\mu_0^J := P\{M_0^{j_1}(1), \cdots, M_0^{j_N}(N)\} = \prod_{l=1}^{N} \mu_0^{j_l}(l). \tag{18}$$

The following notations and definitions are used regarding the measurements at sensor l. Note that, in general, at any time k, some measurements may be due to clutter and some due to the target, i.e. there can be more than a single measurement at time k at sensor l. The measurement set (not yet validated) generated by sensor l at time k is denoted as

$$Z_k^l := \{z_k^{l(1)}, z_k^{l(2)}, \cdots, z_k^{l(m_l)}\} \tag{19}$$

where m_l is the number of measurements generated by sensor l at time k. Variable $z_k^{l(i)}$ $(i = 1, \cdots, m_l)$ is the ith measurement within the set. The validated set of measurements of sensor l at time k will be denoted by Y_k^l, containing $\overline{m}_l$ $(\leq m_l)$ measurement vectors. The cumulative set of validated measurements from sensor l up to time k is denoted as

$$\mathcal{Z}_1^{k(l)} = \{Y_1^l, Y_2^l, \cdots, Y_k^l\}. \tag{20}$$

The cumulative set of validated measurements from all sensors up to time k is denoted as

$$\mathcal{Z}_1^k = \{\mathcal{Z}_1^{k(1)}, \mathcal{Z}_1^{k(2)}, \cdots, \mathcal{Z}_1^{k(q)}\} \tag{21}$$

where q is the number of sensors.

Assuming there are no unresolved measurements (i.e. measurement associated with two or more targets simultaneously), any measurement therefore is either associated with a target or caused by clutter. Our goal is to find the global state estimate

$$\widehat{x}_{k|k} := E\{x_k|\mathcal{Z}_1^k\} \tag{22}$$

and the associated error covariance matrix

$$P_{k|k} = E\{[x_k - \widehat{x}_{k|k}][x_k - \widehat{x}_{k|k}]'|\mathcal{Z}_1^k\} \tag{23}$$

where x_k' denotes the transpose of x_k. Included in the above formulation is state estimates of individual targets.

3. IMM/JPDA COUPLED FILTERING ALGORITHM

We now modify the IMM/JPDA algorithm of [10] to apply to the coupled system (4)-(18); it will be called IMM/JPDACF (CF stands for coupled filter). The approach of [10], in turn, is based on the approaches of [15], [13], [6] and [3]. As in [15] and [10], for convenience, we confine our attention to the case of 2 sensors; however, the algorithm can be easily adapted to the case of arbitrary q sensors. As the IMM/MSPDAF algorithm is well-explained in [15] and Sec. 4.5 of [6], the JPDAF algorithm is well-explained in Sec. 6.2 of [6] and Sec. 9.3 of [3] and the IMM/JPDA filter is given in detail in [10] (where all the underlying assumptions and approximations may be found in further detail), we will only briefly outline the basic steps in "one cycle" (i.e. processing needed to update for a new set of measurements) of the IMM/JPDA coupled filter.

Assumed available: Given the state estimate $\widehat{x}_{k-1|k-1}^J := E\{x_{k-1}|M_{k-1}^J, \mathcal{Z}_1^{k-1}\}$, the associated covariance $P_{k-1|k-1}^J$ and the conditional mode probability $\mu_{k-1}^J = P[M_{k-1}^J|\mathcal{Z}_1^{k-1}]$ at time $k-1$ for each global mode $J \in \bar{\mathcal{M}}_n := \mathcal{M}_n \times \cdots \times \mathcal{M}_n$.

Step 3.1. Interaction – mixing of the estimate from the previous time $(\forall J \in \bar{\mathcal{M}}_n)$:

predicted mode probability:

$$\mu_k^{J-} := P\{M_k^J|\mathcal{Z}_1^{k-1}\} = \sum_I p_{IJ}\mu_{k-1}^I. \tag{24}$$

mixing probability:

$$\mu^{I|J} := P\{M_{k-1}^I|M_k^J, \mathcal{Z}_1^{k-1}\} = p_{IJ}\mu_{k-1}^I/\mu_k^{J-}. \tag{25}$$

mixed estimate:

$$\widehat{x}_{k-1|k-1}^{0J} := E\{x_{k-1}|M_k^J, \mathcal{Z}_1^{k-1}\} = \sum_I \widehat{x}_{k-1|k-1}^I \mu^{I|J}. \tag{26}$$

covariance of the mixed estimate:

$$P^{0J}_{k-1|k-1} := E\{[x_{k-1}-\widehat{x}^{0J}_{k-1|k-1}][x_{k-1}-\widehat{x}^{0J}_{k-1|k-1}]'|M^J_k, Z^{k-1}_1\}$$

$$= \sum_I \{P^I_{k-1|k-1}+[\widehat{x}^I_{k-1|k-1}-\widehat{x}^{0J}_{k-1|k-1}][\widehat{x}^I_{k-1|k-1}-\widehat{x}^{0J}_{k-1|k-1}]'\}\mu^{I|J}. \tag{27}$$

Step 3.2. Predicted state and measurements for Sensor 1 $(\forall J \in \mathcal{M}_n)$:
State prediction:

$$\widehat{x}^J_{k|k-1} := E\{x_k|M^J_k, Z^{k-1}_1\} = F^J_{k-1}\widehat{x}^{0J}_{k-1|k-1}. \tag{28}$$

State prediction error covariance:

$$P^J_{k|k-1} = E\{[x_k - \widehat{x}^J_{k|k-1}][x_k - \widehat{x}^J_{k|k-1}]'|M^J_k, Z^{k-1}_1\}$$

$$= F^J_{k-1}P^{0J}_{k-1|k-1}F^{J'}_{k-1} + G^J_{k-1}Q^J_{k-1}G^{J'}_{k-1}. \tag{29}$$

Using (2) and (28), the global mode-conditioned predicted measurement for sensor 1 is

$$\widehat{z}^{J,1}_k := h^{J,1}(\widehat{x}^J_{k|k-1}). \tag{30}$$

Using the linearized version (14), the covariance of the mode-conditioned residual $\nu^{J,1(I)}_k := z^{1(I)}_k - \widehat{z}^{J,1}_k$, $(z^{1(I)}_k := \mathrm{col}\{z^{1(i_1)}_k, \cdots, z^{1(i_N)}_k\})$, is given by

$$S^{J,1}_k := E\{\nu^{J,1(I)}_k \nu^{j,1(I)\prime}_k|M^J_k, Z^{k-1}_1\} = H^{J,1}_k P^J_{k|k-1}H^{J,1\prime}_k + R^{J,1}_k \tag{31}$$

where $H^{J,1}_k$ is the first order derivative (Jacobian matrix) of $h^{J,1}(.)$ at $\widehat{x}^{J(0)}_{k|k-1}$. Note that (31) assumes that $z^{1(i_r)}_k$ originates from the target r; the result (31) does not depend upon the actual measurements.

Step 3.3. Measurement validation for sensor 1 $(\forall j \in \mathcal{M}_n)$: There is uncertainty regarding the measurements' origins. Therefore, we perform validation for each target separately. There are two steps to measurement validation. First perform measurement validation for each target r $(r \in \mathcal{T}_N)$ separately. For target r, the validation region is taken to be the same for all models, i.e., as the largest of them. Let $S^{j,1}_k(r)$ denote the $n_{z1} \times n_{z1}$ submatrix of $S^{J,1}_k$ including the rows and columns of the latter numbered as $(r-1)n_{z1}+m$, $m = 1, 2, \cdots, r$. That is, $S^{j,1}_k(r)$ is based on the information relevant to target r only. Let $\widehat{z}^{j_r,1}_k(r)$ denote the $n_{z1} \times 1$ sub-column of $\widehat{z}^{J,1}_k$ including the rows of the latter numbered as $(r-1)n_{z1}+m$, $m = 1, 2, \cdots, r$; that is, $\widehat{z}^{j_r,1}_k(r)$ is the mode-conditioned predicted measurement of target r for sensor 1. Let $(|A| = \det(A))$

$$j_r := \arg\left\{\max_{j\in\mathcal{M}_n} |S^{j,1}_k(r)|\right\}. \tag{32}$$

Then measurement $z^{1(i)}_k$ ($i = 1, 2, \cdots, m_1$) is validated if and only if

$$[z^{1(i)}_k - \widehat{z}^{j_r,1}_k(r)]'[S^{j_r,1}_k(r)]^{-1}[z^{1(i)}_k - \widehat{z}^{j_r,1}_k(r)] < \gamma \tag{33}$$

where γ is an appropriate threshold. The volume of the validation region with the threshold γ is

$$V^1_k(r) := c_{n_{z1}}\gamma^{n_{z1}/2}|S^{j_r,1}_k(r)|^{1/2} \tag{34}$$

where n_{z1} is the dimension of the measurement and $c_{n_{z1}}$ is the volume of the unit hypersphere of this dimension ($c_1 = 2, c_2 = \pi, c_3 = 4\pi/3$, etc.). Choice of γ is discussed in more detail in ([6], Sec. 2.3.2) (see also Sec. 4. later). After performing the validation for each target separately, the volume of validation region for the whole target set is approximated by

$$V^1_k = \sum_{r=1}^{N} V^1_k(r). \tag{35}$$

Step 3.4. State estimation with validated measurements from sensor 1 $(\forall J \in \mathcal{M}_n)$: From among all the raw measurements from sensor 1 at time k, i.e., $Z^1_k := \{z^{1(1)}_k, z^{1(2)}_k, \cdots, z^{1(m_1)}_k\}$, define the set of validated measurement for sensor 1 at time k as

$$Y^1_k := \{y^{1(1)}_k, y^{1(2)}_k, \cdots, y^{1(\overline{m}_1)}_k\} \tag{36}$$

where $\overline{m}_1$ is total number of validated measurement for sensor 1 at time k. and

$$y^{1(i)}_k := z^{1(l_i)}_k \tag{37}$$

where $1 \leq l_1 < l_2 < \cdots < l_{\overline{m}_1} \leq m_1$ when $\overline{m}_1 \neq 0$. Note that all targets share a common validated measurement set Y^1_k from sensor 1.

We now consider joint probabilistic data association across targets following [6] and [3], but for global target state. A marginal association event θ_{ir} is said to be effective at time k when the validated measurement $y^{1(i)}_k$ is associated with (i.e. originates from) target r ($r = 0, \cdots, N$ where $r = 0$ means that the measurement is caused by clutter). Assuming that there are no unresolved measurements, a joint association event Θ is effective when a set of marginal events $\{\theta_{ir}\}$ holds true simultaneously. That is, $\Theta = \cap_{i=1}^{\overline{m}_1}\theta_{ir_i}$ where r_i is the index of the target to which measurement $y^{1(i)}_k$ is associated in the event under consideration. Define the validation matrix (as in [6] and [3])

$$\Omega = [\omega_{ir}] \quad i = 1, \cdots, \overline{m}_1, \quad r = 0, \cdots, N \tag{38}$$

where $\omega_{ir} = 1$ if the measurement i lies in the validation gate of target r, else it is zero. A joint association event Θ is represented by the event matrix

$$\widehat{\Omega}(\Theta) = [\widehat{\omega}_{ir}(\Theta)] \quad i = 1, \cdots, \overline{m}_1, \quad r = 0, \cdots, N \tag{39}$$

where

$$\widehat{\omega}_{ir}(\Theta) = \begin{cases} 1 & \text{if } \theta_{ir} \subset \Theta \\ 0 & \text{otherwise.} \end{cases} \tag{40}$$

A feasible association event is one where a measurement can have only one source

$$\sum_{r=0}^{N} \widehat{\omega}_{ir}(\Theta) = 1 \quad \forall i, \tag{41}$$

and where at most one measurement can originate from a target

$$\delta_r(\Theta) := \sum_{i=0}^{\overline{m}_1} \widehat{\omega}_{ir}(\Theta) \leq 1 \quad \text{for } r = 1, \cdots, N. \tag{42}$$

The above joint events Θ are mutually exclusive and exhaustive. Following the definitions in [6] and [3], define the binary measurement association indicator

$$\tau_i(\Theta) := \sum_{r=1}^{N} \widehat{\omega}_{ir}(\Theta), \quad i = 1, \cdots, \overline{m}_1, \tag{43}$$

to indicate whether the validated measurement $y^{1(i)}_k$ is associated with a target in event Θ. Further, the number of false (unassociated) measurements in event Θ is

$$\phi(\Theta) = \sum_{i=1}^{\overline{m}_1} [1 - \tau_i(\Theta)]. \tag{44}$$

We will limit our discussion to nonparametric JPDA [6]. One can evaluate the likelihood that the global mode is J as

$$\Lambda_k^{J,1} := p[Y_k^1|M_k^J, Z_1^{k-1}]$$
$$= \sum_{\Theta} p[Y_k^1|\Theta, M_k^J, Z_1^{k-1}]P\{\Theta|M_k^J, Z_1^{k-1}\}$$
$$= \sum_{\Theta} p[Y_k^1|\Theta, M_k^J, Z_1^{k-1}]P\{\Theta\} \tag{45}$$

where, as in (9-31) of [3], the irrelevant conditioning terms have been omitted in the last line of (45) and the conditioning on $\overline{m}_1$ is implicit in the event Θ. The second term (apriori joint association probabilities) in the last line of (45) turns out to be (Sec. 6.2 of [6], Sec. 9.3 of [3])

$$P\{\Theta\} = \frac{\phi(\Theta)!\,\epsilon}{\overline{m}_1!}\prod_{s=1}^{N}(P_D)^{\delta_s(\Theta)}(1-P_D)^{1-\delta_s(\Theta)} \tag{46}$$

where P_D is the detection probability at sensor 1 (assumed to be the same for all targets) and $\epsilon > 0$ is a "diffuse" prior (for nonparametric modeling of clutter) whose exact value is irrelevant. Unlike [10], we do not assume that the states of the targets (including the modes) conditioned on the past observations are mutually independent. Then the first term in the last line of (45) can be written as

$$p[Y_k^1|\Theta, M_k^J, Z_1^{k-1}] = V_1^{-\phi(\Theta)}p[\tilde{Y}_k^1(\Theta)|M_k^J, Z_1^{k-1}] \tag{47}$$

where $\tilde{Y}_k^1(\Theta) \subset Y_k^1$ is a subset of the validated measurements Y_k^1, consisting of the measurements associated with the targets as specified by Θ. The number of measurements in $\tilde{Y}_k^1(\Theta)$ equal $\overline{m}_1 - \phi(\Theta)$ where $\phi(\Theta)$ is the number of false alarms.

Define a $\overline{m}_1 \times [\overline{m}_1 - \phi(\Theta)]$ matrix $\underline{\widehat{\Omega}}(\Theta)$ as a submatrix of $\widehat{\Omega}(\Theta)$ obtained by deleting the first column and all null columns of $\widehat{\Omega}(\Theta)$. Then for a given Θ, we have a measurement vector $\tilde{Y}_k^1(\Theta)$ of dimension $(\sum_{i=1}^{\overline{m}_1}\tau_i(\Theta))n_{z1}$ given by

$$\tilde{Y}_k^1(\Theta) = (I_{n_{z1}} \otimes \underline{\widehat{\Omega}}'(\Theta))\mathrm{col}\{y_k^{1(i)},\ i = 1, 2, \cdots, \overline{m}_1\} \tag{48}$$

where we stack up all target-associated validated measurements in Θ in ascending order of targets, I_n is the $n \times n$ identity matrix, and the symbol $\otimes$ denotes the Kronecker product. Define a $[(\overline{m}_1 - \phi(\Theta))n_{z1}] \times [Nn_x]$ matrix $H_k^{J,1}(\Theta)$ as a submatrix of $H_k^{J,1}$ (see (15)) obtained by deleting all i-th block rows ($n_{z1} \times .$) of $H_k^{J,1}$ for which $\delta_i(\Theta) = 0$. That is, we have modified (15) to keep only the block elements associated with target-associated measurements in Θ. It then follows that the linearized measurement equation for $\tilde{Y}_k^1(\Theta)$ is given by

$$\tilde{Y}_k^1(\Theta) = H_k^{J,1}(\Theta)x_k + w_k^{J,1}. \tag{49}$$

Conditioned on the joint association event Θ and mode J, the "coupled" innovations is given by

$$\nu_k^{J,1}(\Theta) = \begin{cases} \tilde{Y}_k^1(\Theta) - \widehat{z}_k^{J,1}(\Theta) & \text{if } \delta_r(\Theta) = 1 \text{ for some } r \in \{1, 2, \cdots, N\}, \\ 0 & \text{otherwise,} \end{cases} \tag{50}$$

where $\widehat{z}_k^{J,1}(\Theta)$ is a subvector of (30) obtained by deleting all i-th block rows ($n_{z1} \times 1$) of $\widehat{z}_k^{J,1}$ for which $\delta_i(\Theta) = 0$. The conditional pdf (probability density function) of the validated measurements $\tilde{Y}_k^1(\Theta)$ given their origins (specified by Θ) and the global target mode J, is given by

$$p[\tilde{Y}_k^1(\Theta)|M_k^J, Z_1^{k-1}] = \mathcal{N}(\tilde{Y}_k^1(\Theta); \widehat{z}_k^{J,1}(\Theta), S_k^{J,1}(\Theta)) \tag{51}$$

where

$$\mathcal{N}(x; y, P) := |2\pi P|^{-1/2}\exp[-\frac{1}{2}(x-y)'P^{-1}(x-y)] \tag{52}$$

and

$$S_k^{J,1}(\Theta) := E\{\nu_k^{J,1}(\Theta)\nu_k^{J,1}(\Theta)'|M_k^J, Z_1^{k-1}\}$$
$$= H_k^{J,1}(\Theta)P_{k|k-1}^J H_k^{J,1}(\Theta)' + R_k^{J,1}. \tag{53}$$

The probability of the joint association event Θ given that global mode J is effective from time $k-1$ through k is

$$\beta_k^{J,1}(\Theta) := P\{\Theta|M_k^J, Z_1^{k-1}, Y_k^1\}$$
$$= \frac{1}{c}p[Y_k^1|\Theta, M_k^J, Z_1^{k-1}]P\{\Theta|M_k^J, Z_1^{k-1}\} = \frac{1}{c}p[Y_k^1|\Theta, M_k^J, Z_1^{k-1}]P\{\Theta\} \tag{54}$$

where the first term can be calculated from (47)-(53), the second term from (46), and c is a normalization constant such that $\sum_\Theta P\{\Theta|M_k^J, Z_1^{k-1}, Y_k^1\} = 1$.

Using $\widehat{x}_{k|k-1}^J$ (from (28)) and its covariance $P_{k|k-1}^J$ (from (29)), one computes the partial update $\widehat{x}_{k|k}^{J,1}$ and its covariance $P_{k|k}^{J,1}$ following the standard PDAF [6], [3], except that the global state is conditioned on Θ, not the marginal events θ_{ir}; details follow.

$$\text{Kalman gain:}\quad W_k^J(\Theta) = P_{k|k-1}^J H_k^{J,1}(\Theta)'[S_k^{J,1}(\Theta)]^{-1}. \tag{55}$$

Partial update of the state estimate:

$$\widehat{x}_{k|k}^{J,1}(\Theta) := E\{x_k|\Theta, M_k^J, Z_1^{k-1}, Y_k^1\}$$
$$= \begin{cases} \widehat{x}_{k|k-1}^J + W_k^J(\Theta)\nu_k^{J,1}(\Theta) & \text{if } \delta_r(\Theta) \neq 0 \text{ for some } 1 \le r \le N \\ \widehat{x}_{k|k-1}^J & \text{if } \delta_r(\Theta) = 0\ \forall r \in \{1, 2, \cdots, N\}. \end{cases} \tag{56}$$

$$\widehat{x}_{k|k}^{J,1} := E\{x_k|M_k^J, Z_1^{k-1}, Y_k^1\} = \sum_{\Theta}\beta_k^{J,1}(\Theta)\widehat{x}_{k|k}^{J,1}(\Theta) \tag{57}$$

Covariance of $\widehat{x}_{k|k}^{J,1}$:

$$P_{k|k}^{J,1} = P_{k|k-1}^J - \sum_{\Theta:\Theta\neq\Theta_0}\beta_k^{J,1}(\Theta)W_k^J(\Theta)S_k^{J,1}(\Theta)W_k^J(\Theta)'$$
$$+ \sum_{\Theta}\beta_k^{J,1}(\Theta)W_k^J(\Theta)\nu_k^{J,1}(\Theta)\nu_k^{J,1}(\Theta)'W_k^J(\Theta)'$$
$$- \left[\sum_{\Theta}\beta_k^{J,1}(\Theta)W_k^J(\Theta)\nu_k^{J,1}(\Theta)\right]\left[\sum_{\Theta}\beta_k^{J,1}(\Theta)W_k^J(\Theta)\nu_k^{J,1}(\Theta)\right]' \tag{58}$$

where Θ_0 denotes Θ for which $\delta_r(\Theta) = 0\ \forall r \in \{1, 2, \cdots, N\}$. Eqn. (58) follows in a manner similar to eqn. (3.4.2-10) in [6].

Step 3.5. The mode-conditioned predicted measurements for sensor 2 ($\forall J \in \mathcal{M}_n$): Using (2) and (57), the "predicted" measurement for sensor 2 is given by

$$\widehat{z}_k^{J,2} := h^{J,2}(\widehat{x}_{k|k}^{J,1}). \tag{59}$$

Using the linearized version (3), the covariance of the global mode-conditioned residual $\nu_k^{J,2(I)} := z_k^{2(I)} - \widehat{z}_k^{J,2}$ is given by

$$S_k^{J,2} := E\{\nu_k^{J,2(I)}\nu_k^{J,2(I)\prime}|M_k^J, Z_1^{k-1}, Y_k^1\}$$
$$= H_k^{J,2}P_{k|k}^{J,1}H_k^{J,2\prime} + R_k^{J,2} \tag{60}$$

where $H_k^{J,2}$ is the first order derivative (Jacobian matrix) of $h^{J,2}(.)$ at $\widehat{x}_{k|k}^{J,1}$.

Step 3.6. Measurement validation for sensor 2: This is similar to Step 3.3 where we replace $S_k^{J,1}$ with $S_k^{J,2}$, $z_k^{1(i)}$ with $z_k^{2(i)}$, m_1 with m_2, $V_k^1(r)$ with $V_k^2(r)$, and V_k^1 with V_k^2. Details are similar to that in Step 3.3, hence are omitted.

Step 3.7. Update with validated measurements for sensor 2 ($\forall J \in \mathcal{M}_n$): This step is similar to Step 3.4. Using the validated measurements obtained from Step 3.6 and starting from $\hat{x}^{J,1}_{k|k}$ and $P^{J,1}_{k|k}$, one computes the final updates $\hat{x}^{J}_{k|k}$ and $P^{J}_{k|k}$, and the likelihood

$$\Lambda^{J,2}_k := p[Y^2_k | M^J_k, Y^1_k, Z^{k-1}_1]. \quad (61)$$

The details are similar to that in Step 3.4, hence are omitted.

Step 3.8. Update of mode probabilities ($\forall j \in \mathcal{M}_n$, $\forall r \in \mathcal{T}_N$):

$$\mu^J_k := P[M^J_k | Z^k_1] = \frac{1}{c} \mu^{J-}_k \Lambda^{J,1}_k \Lambda^{J,2}_k \quad (62)$$

where c is a normalization constant such that $\sum_J \mu^J_k = 1$. For individual targets we have

$$\mu^{j_r}_k(r) := P[M^{j_r}_k(r) | Z^k_1]$$

$$= \sum_{j_1=1}^{n} \cdots \sum_{j_{r-1}=1}^{n} \sum_{j_{r+1}=1}^{n} \cdots \sum_{j_N=1}^{n} \mu^{j_1,\cdots,j_{r-1},j_r,j_{r+1},\cdots,j_N}_k \quad (63)$$

with $J = (j_1, \cdots, j_N)$ in (62).

Step 3.9. Combination of the mode-conditioned estimates ($\forall r \in \mathcal{T}_N$): The final global state estimate update at time k is given by

$$\hat{x}_{k|k} = \sum_J \hat{x}^J_{k|k} \mu^J_k \quad (64)$$

and its covariance is given by

$$P_{k|k} = \sum_J \left\{ P^J_{k|k} + [\hat{x}^J_{k|k} - \hat{x}_{k|k}][\hat{x}^J_{k|k} - \hat{x}_{k|k}]' \right\} \mu^J_k. \quad (65)$$

The state estimate $\hat{x}_{k|k}(r)$ for target r is the n_x-subvector of $\hat{x}_{k|k}$ consisting of elements $(r-1)n_x + m$, $m = 1, 2, \cdots, n_x$.

Remark 1. Compared to the uncoupled filtering of [10] where the equations are formulated conditioned on marginal association events θ_{ir}, here we have conditioning on joint association events Θ for coupled filtering. Eqn. (51) does not decompose into the product of marginal probabilities as in [10].

4. SIMULATION EXAMPLE

We now consider tracking two highly maneuvering targets in clutter. We illustrate the proposed filtering algorithm via this example.

The True Trajectory: Target 1 starts at location [21689 10840 40] in Cartesian coordinates in meters. The initial velocity (in m/s) is [−8.3 − 399.9 0] and the target stays at constant altitude with a constant speed of 400m/s. Its trajectory is: a straight line with constant velocity between 0 and 17s, a coordinated turn (0.15 rad/s) with constant acceleration of 60 m/s^2 between 17 and 30s, a straight line with constant velocity between 30 and 55s, a coordinated turn (0.1 rad/s) with constant acceleration of 40 m/s^2 between 55 and 70s, and a straight line with constant velocity between 70 and 87s. Target 2 starts at location [30000 − 3040 40] in Cartesian coordinates in meters. The initial velocity (in m/s) is [−382 157 0] and the target stays at constant altitude with a constant speed of 413m/s. Its trajectory is: a straight line with constant velocity between 0 and 44s, a coordinated turn (0.075 rad/s) with constant acceleration of 30 m/s^2 between 44 and 59s, and a straight line constant velocity between 59 and 87s.

The Target Motion Models: These are patterned after [15]. The motion models for the two targets are identical. In each mode the target dynamics are modeled in Cartesian coordinates as $x_k(r) = F(r)x_{k-1}(r) + G(r)v_{k-1}(r)$ where the state of the target is position, velocity and acceleration in each of the 3 Cartesian coordinates (x, y and z). Thus $x_k(r)$ is of dimension 9 (n_x=9) for each target. Three models are considered in the following discussion. They are exactly as in [10] to which one is refered for more details. The initial model probabilities for two targets are identical: $\mu^1_0 = 0.8$, $\mu^2_0 = 0.1$ and $\mu^3_0 = 0.1$. The mode switching probability matrix for two targets is also identical and is as in [10].

The Sensors: Two sensors (we assume collocation, and time synchronization of observations, etc.) are used to obtain the measurements. The measurements from sensor l for model j are $z^l_k = h^{j,l}(x_k) + w^{j,l}_k$, $l = 1, 2$, reflecting range and azimuth angle for sensor 1 (radar), and azimuth and elevation angles for sensor 2 (infrared). For further details, see [10]. The measurement noise $w^{j,l}_k$ for sensor l is assumed to be zero-mean white Gaussian with known covariances $R^1 = \mathrm{diag}[q_r, q_{a1}] = \mathrm{diag}[400\mathrm{m}^2,\ 49\mathrm{mrad}^2]$ with q_{a1} and q_r denoting the variances for the radar azimuth and range measurement noises, respectively, and $R^2 = \mathrm{diag}[q_{a2}, q_e] = \mathrm{diag}[4\mathrm{mrad}^2,\ 4\mathrm{mrad}^2]$ with q_{a2} and q_e denoting the variances for the infrared sensor azimuth and elevation measurement noises, respectively. Both sensors are assumed to be located at the coordinate system origin. The sampling interval was $T = 1$s and it was assumed that the probability of detection $P_D = 0.997$ for both sensors.

The Clutter: For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of $\lambda_1 = 20 \times 10^{-6}$/m mrad for sensor 1 and $\lambda_2 = 2 \times 10^{-4}$/mrad^2 for sensor 2. These statistics were used for generating the clutter in all simulations. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

Other Parameters: The gates for setting up the validation regions for both the sensors were based on the threshold $\gamma = 16$. With the measurement vector of dimension 2, this leads to a gate probability $P_G = 0.9997$ (see p. 96 of [6]).

Simulation Results: The results were obtained from 100 Monte Carlo runs. Fig. 1 shows the true trajectory of the two targets and the distance between the two targets as a function of time. The two targets start out far apart, move close to each other from 30 to 45 seconds, and then move apart again. Fig. 2 shows the results of the proposed IMM/JPDACF based on 100 runs. Fig. 3 shows the results of the uncoupled IMM/JPDAF of [10] based on 100 runs. It is seen from Fig. 3 that there is a loss of track (target swapping occurs in 2 out of 100 runs).

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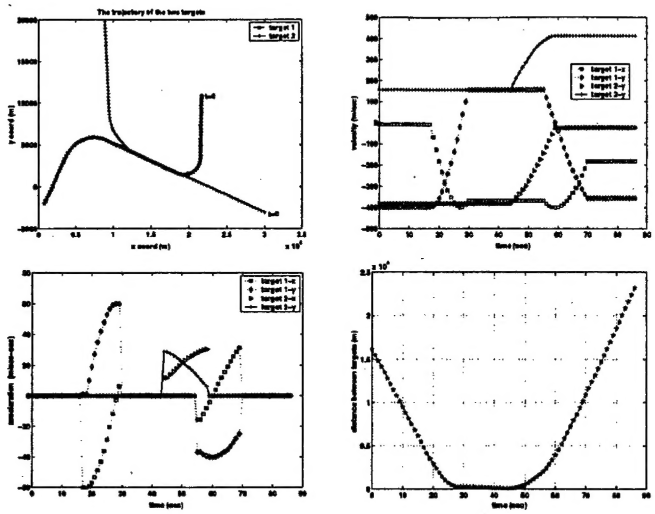

Figure 1. The true trajectories of the maneuvering targets (read left-to-right, top-to-bottom): (a) Position in the xy–plane. (b) x and y velocities. (c) x and y accelerations. (d) distance between the targets.

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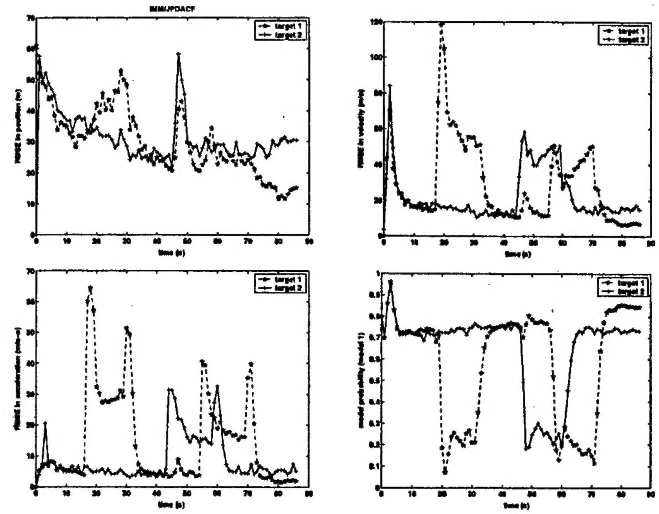

Figure 2. Performance of the proposed IMM/JPDACF based on 100 runs (read left to right, top to bottom): (a) RMSE in position. (b) RMSE in velocity. (c) RMSE in acceleration. (d) CV model probability $P[M_k^1(r)|Z_1^k]$ for $r = 1, 2$. (RMSE = root mean-square error; CV = constant velocity)

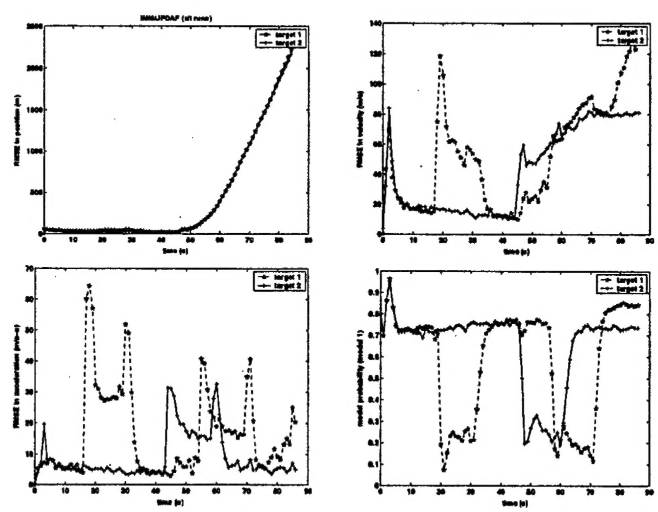

Figure 3. Performance of the IMM/JPDAF of [10] based on 100 runs (read left to right, top to bottom): (a) RMSE in position. (b) RMSE in velocity. (c) RMSE in acceleration. (d) CV model probability $P[M_k^1(r)|Z_1^k]$ for $r = 1, 2$. (RMSE = root mean-square error; CV = constant velocity)